Measurements of newly defined intensimetric quantities and their physical interpretation

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(Received 26 November 1996; revised 9 June 1997; accepted 16 June 1997)

This paper deals with the measurement and physical interpretation of the quantities **a** and **r** introduced in a previous paper [D. Stanzial, N. Prodi, and G. Schiffrer, ''Reactive intensity for general fields and energy polarization,'' J. Acoust. Soc. Am. **99**, 1868–1876 (1996)]. The quantity **a**, which has the same time dependence as the squared acoustic pressure and the same direction as the time averaged sound intensity $\langle \mathbf{j} \rangle$, will be called here ''radiating'' intensity, while **r** which has zero average, will be called ''oscillating'' intensity. A coherent picture of the energy transfer process in steady sound fields based on the decomposition $\mathbf{j}=\mathbf{a}+\mathbf{r}$ of the instantaneous sound intensity will be sketched and discussed. Furthermore, a direct experimental comparison between **a** and **r** and the real and imaginary parts of the complex intensity is presented for some field conditions. © *1997 Acoustical Society of America.* [S0001-4966(97)00510-9]

PACS numbers: 43.20.Ye, 43.20.Rz, 43.58.Fm [JEG]

INTRODUCTION

Since the advent of instrumentation devoted to the measure of acoustic intensity, it has been possible to represent the energy transfer process within sound fields by the use of experimentally measured intensity maps. A former noticeable result in this respect was due to Waterhouse and Yates,[1] whose description of the energy transfer process was developed on the concept of ''energy streamlines'': that is, lines along which all the energy in the field did flow. These lines are tangent to the average acoustic intensity at every point inside the sound field.

An early criticism to this picture came from Mann *et al.*[2] who started their analysis from the decomposition of instantaneous intensity **j** into active and reactive parts and investigated, for a pure-tone field, how the decomposition affected the energy transfer process description. Even if the term containing the reactive intensity time averages to zero, they rightly concluded that both the active and reactive intensities contribute to the actual energy flux. Another topic of their contribution was the introduction of ''sources'' for reactive intensity, which are formally related to the Lagrangian of the field, but whose physical significance is still an unresolved problem.[3]

The definition of complex intensity for general fields as found in Ref. 4 leads to the conclusion that, since the time average of reactive intensity is not vanishing, then in order to consider all the contributions to the energy transfer process, it is necessary to also include, together with the active intensity vector field, the flow of energy along the ''paths'' of averaged reactive intensity. In other words, the generalization of the point of view of Mann *et al.* is that a time average picture of the energy transfer process is only achieved by means of two vector fields.

All the above-mentioned interpretations present a critical aspect: the decomposition of instantaneous intensity **j**. Once the mathematical properties of the decomposition are well-defined, then a consequent picture of the energy transfer process can be developed accordingly.

A new contribution to this matter was presented in recent years in Refs. 5 and 6 where the general decomposition $\mathbf{j}=\mathbf{a}+\mathbf{r}$ has been introduced for stationary fields. In Refs. 5 and 6 **a** and **r** have been called ''instantaneous active'' and ''reactive'' intensity. Here, these quantities will be called respectively *radiating* and *oscillating* instantaneous intensity, in order to avoid confusion with the real and imaginary parts of the complex intensity.[4] The mathematical features of such decomposition will be summarized below; here we simply note that the time averaged value of radiating intensity equals the active intensity, while the time averaged value of oscillating intensity is zero by definition.

On the basis of this decomposition, a framework will be developed which gives both a time average and a time-dependent interpretation of the energy transfer process within general stationary fields. In the time average framework only one vector field (averaged radiating intensity) can be detected, which is tangent to streamlines of energy. But, different from Waterhouse's first intuition, not all of the energy actually flows along these streamlines. In fact, there is also energy which is transferred by the oscillating part of intensity, which cannot contribute to the net flow because it must oscillate due to the vanishing of time average value of **r**. Furthermore, within the proposed picture of energy transfer, plane progressive waves are considered as ideal guides for energy radiation and, conversely, standing waves as sound fields where energy is locally oscillating.

The utility of building up such a framework for the description of the time averaged energy transfer process is evident. On the basis of this picture, for instance, practically all the intensity maps so far collected in the more disparate sectors of applied acoustics should be easily reinterpreted: The only lack affecting this kind of sound field visualization can be filled by adding to the intensity maps a suitable measure telling how much energy oscillates around every measuring point.

In this paper the above outlined framework for the energy transfer process is sketched and newly defined instantaneous and time averaged intensimetric quantities are introduced and their measures are reported. A comparison of these quantities with the active and reactive intensities known from literature is then accomplished and their different behavior is analyzed to some extent.

I. DEFINITIONS OF INSTANTANEOUS INTENSITIES

A. Definition of new intensimetric quantities

The behavior of the instantaneous intensity vector $\mathbf{j}:=p\mathbf{v}$ can be understood by analyzing the time dependence of pressure p and velocity $\mathbf{v}$ of the air particle at the measurement point. If pressure and velocity have the same time dependence, their values cross the respective zero reference line at the same instant and thus the two quantities yield values with the same algebraic sign. Then the components of their product are positive and its time average is nonvanishing. This occurrence means that a nonzero *time averaged* momentum is transmitted to the confining air particles in such a way that energy flux streamlines are built-up all over the sound field. On the other hand, when pressure and velocity do not cross the zero line at the same instant, a fraction of the sound energy does not flow along the mean streamline passing through the given point and oscillates locally.

These two fundamental features of energy transfer mechanism are instantaneously described by the novel intensimetric quantities $\mathbf{a}$ and $\mathbf{r}$ introduced in Ref. 5.

Part $\mathbf{a}$ is responsible for energy propagation along the streamlines and can be called *radiating intensity*; in formulas it reads:

$$\mathbf{a}(\mathbf{x},t):=\frac{p^2\langle p\mathbf{v}\rangle}{\langle p^2\rangle}, \tag{1}$$

where $\langle\cdot\rangle$ stands for the stationary time averaging procedure. The time averaged radiating intensity coincides with the well-known active intensity

$$\mathbf{A}(\mathbf{x}):=\langle\mathbf{a}\rangle=\langle p\mathbf{v}\rangle.$$

Putting aside mathematical details it can be now noted that the complement to instantaneous intensity which is achieved by a simple vector subtraction:

$$\mathbf{r}(\mathbf{x},t):=\mathbf{j}(\mathbf{x},t)-\mathbf{a}(\mathbf{x},t)=\frac{\langle p^2\rangle p\mathbf{v}-p^2\langle p\mathbf{v}\rangle}{\langle p^2\rangle} \tag{2}$$

averages to zero by definition, i.e., $\langle\mathbf{r}\rangle\equiv 0$.

The vector quantity $\mathbf{r}$ which, differently from $\mathbf{a}$, must change its orientation at a fixed point, can be called *oscillating intensity*. Since the first-order statistical moment (time average) of the vector $\mathbf{r}$ always vanishes, a nontrivial measure of the average properties of $\mathbf{r}$ has to be obtained by the calculation of the first nonvanishing higher-order moment which, in this case, is the second-order one. The suitable mathematical entity to consider, in order to describe the second-order statistical properties of a vector, is thus the second-order tensor $\mathscr{R}=\sqrt{2\langle\mathbf{r}\otimes\mathbf{r}\rangle}$. This tensor summarizes the most important time averaged spatial informations concerning oscillations of energy. In Ref. 6 a thorough exposition of the tensor formalism can be found.

For the aim of this paper we shall only consider the scalar value

$$R:=\sqrt{2\langle\mathbf{r}^2\rangle}, \tag{3}$$

which is directly obtained as the trace of the former tensor.

B. Alternative definitions

Mathematically speaking, the definition of radiating intensity consists in a multiplication of each component of active intensity by a normalized scalar function $p^2/\langle p^2\rangle$ which clearly has the same time dependence as the instantaneous squared pressure. Since both pressure and the three components of velocity are solutions of the wave equation, there is no physically sensible reason to prefer pressure to the velocity components in the definition given above. Hence, instead of squared pressure, one may equally pursue an alternative normalization based on the square of the three components of velocity. This procedure involves the multiplication of each component of active intensity by a normalization scalar function derived from the respective squared velocity component. On this basis a pair of alternative definitions can be introduced for radiating and oscillating intensities, which will be indicated by $\mathbf{a}'$ and $\mathbf{r}'$:

$$\mathbf{a}'(\mathbf{x},t):=\begin{pmatrix} a'_x \\ a'_y \\ a'_z \end{pmatrix}; \quad a'_i:=\frac{v_i^2\langle pv_i\rangle}{\langle v_i^2\rangle}; \quad i=x,y,z, \tag{4}$$

$$\mathbf{r}'(\mathbf{x},t):=\mathbf{j}(\mathbf{x},t)-\mathbf{a}'(\mathbf{x},t). \tag{5}$$

A few remarks have to be made, as this alternative definitions might seem to produce results different from those obtained by the definitions given in the previous section. In fact, it clearly appears that, starting from the same $p(t)$ and $\mathbf{v}(t)$, the two formulations will yield different time histories of both radiating and oscillating intensity. Nevertheless, it can be shown that the time average properties of the two pairs $\mathbf{a}$, $\mathbf{a}'$ and $\mathbf{r}$, $\mathbf{r}'$ coincide, no matter the sound field. In other words the following relations hold:

$$\langle\mathbf{a}\rangle=\langle\mathbf{a}'\rangle=\langle p\mathbf{v}\rangle=\mathbf{A}, \tag{6}$$

$$\sqrt{2\langle\mathbf{r}^2\rangle}=\sqrt{2\langle\mathbf{r}'^2\rangle}=R. \tag{7}$$

We will support this statement with experimental results in a following section.

Given that $\mathbf{a}$, $\mathbf{a}'$ and $\mathbf{r}$, $\mathbf{r}'$ present the same behavior to second statistical order, it can be stated that the two alternative definitions fit the same proposed picture.

C. Definition of active and reactive intensities

As reported in Ref. 4 the definitions of active $\mathbf{I}(t)$ and reactive $\mathbf{J}(t)$ instantaneous intensities in a narrow-band field are, respectively, the real and imaginary part of the complex intensity $\mathbf{I}_c(t)$, which is introduced by the implementation of an Hilbert transformation process, here denoted by a caret:

$$\mathbf{I}_c(t):=\tfrac{1}{2}[p(t)+i\hat{p}(t)][\mathbf{v}(t)-i\hat{\mathbf{v}}(t)],$$

$$\mathbf{I}(t):=\mathrm{Re}\{\mathbf{I}_c(t)\}=\tfrac{1}{2}[p(t)\mathbf{v}(t)+\hat{p}(t)\hat{\mathbf{v}}(t)], \tag{8}$$

$$\mathbf{J}(t):=\mathrm{Im}\{\mathbf{I}_c(t)\}=\tfrac{1}{2}[\hat{p}(t)\mathbf{v}(t)-p(t)\hat{\mathbf{v}}(t)]. \tag{9}$$

The time average values of the above quantities can be easily calculated and are generally referred to by omitting the explicit time dependence:

$$\mathbf{I} := \langle p\mathbf{v} \rangle, \quad \mathbf{J} := \langle p\hat{\mathbf{v}} \rangle. \tag{10}$$

The instantaneous active intensity averages to the well-known active intensity and, in this respect, **I** and Eq. (6) give the same time average description of energy flux.

Moreover, according to (10), the time average of the instantaneous reactive intensity in a general sound field is nonvanishing. The most remarkable difference between **J** and the time averaged oscillating intensity, R, Eq. (7) is then evident: while R is a scalar quantity, the reactive intensity **J** is a vector quantity.

II. DISCUSSION

First of all, it must be remarked that summarizing the time averaged properties of oscillating intensity by a scalar quantity provides exactly the suitable measure of that part of energy which does not contribute to the net flow. Since oscillating intensity has no average energy streamlines, what one can do is to characterize the total amount of oscillations of energy by the root mean square value R, although neglecting the spatial distribution of such oscillations. A forward step in the analysis shall involve the tensor formalism, which can tell exactly the average magnitude of oscillations of energy along every given direction. On the other hand, the vector field (1) [as well as the alternative field (4)] always has direction and orientation coinciding with that of the active intensity, the modulus being equal only when time averaged. So the averaged streamlines of this quantity coincide with those of active intensity. This leads to the conjecture that the energy transfer within any general stationary sound field can be exhaustively described by the time averaged properties of the radiating and oscillating intensities.

The relationship between the newly defined quantities and active and reactive intensities is also of interest. The equality of the averaged values **A** and **I** in any sound field has been already stressed. Regarding the oscillating intensity it can be proved (see Ref. 6) that the direction along which oscillations of energy occur coincides with that of **J** in a monochromatic field, but this coincidence gets lost in general fields. This fact has also been briefly investigated experimentally. Furthermore the time histories of the paired quantities (1) and (8) as well as those of (2) and (9) have been experimentally obtained, and their comparison will be reported in the next section.

III. EXPERIMENTAL RESULTS

Intensimetric measurements have been carried out in order to experimentally illustrate the newly defined quantities, to check the frame for the energy transfer process based on them, and to relate them with active and reactive intensities known from the literature.

All the measurements were taken along a single Cartesian component coinciding with the axis of a sound intensity probe, Brüel & Kjær 3591 (with 50-mm spacer). The probe supplied two pressure signals to an FFT Brüel & Kjær 2032 analyzer where they were captured and digitally converted. Then they were passed, via an HP-IB port, to a Hewlett-Packard 9817 computer for subsequent processing. The program running on the computer first implemented Euler's equation to get the time history of the velocity in the mid point between microphones and then, exploiting suitable algorithms, gave both the time histories and average values of the acoustic quantities introduced in Sec. I. The test signals were delivered by a small loudspeaker mounted in an enclosed cabinet.

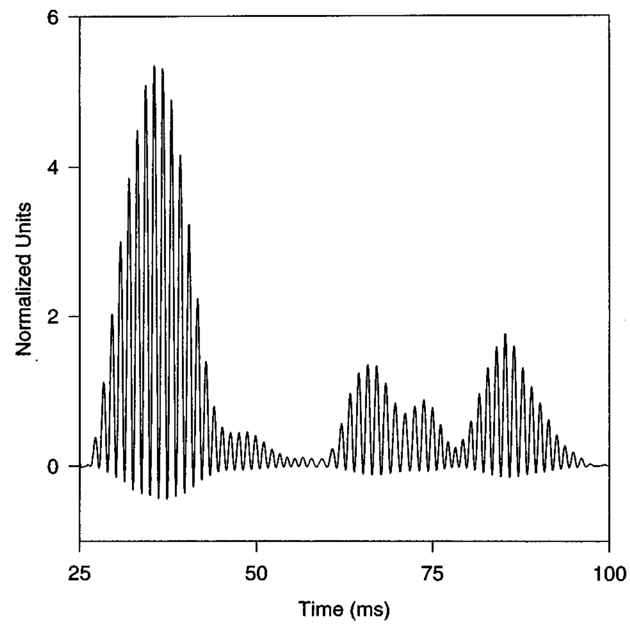

FIG. 1. Instantaneous intensity measured inside an anechoic room. Stimulus is $\frac{1}{3}$ octave band filter noise with center frequency of 400 Hz and the distance between the loudspeaker and the facing probe is 1 m.

The scale normalization for time-dependent intensimetric measurements was accomplished by dividing each time history by the root-mean square of instantaneous intensity (obtained with the same raw data), whereas measures of pressure and velocity were normalized by the root mean square of pressure.

The first set of measurements was carried out in a large anechoic room with the probe facing the loudspeaker at a distance of 1 m. The test signal was noise in a third octave band centered at 400 Hz. In Fig. 1 the instantaneous intensity is reported as reference: We note that a kind of amplitude modulation seems to characterize the plot, showing also a fundamental of twice the center frequency of the band. Figure 2 presents the temporal evolution of Eqs. (1) and (8) for this field. As expected by the given definition, radiating intensity cannot change its sign along the plot as it must always be equal to that of its average **A**. The quantity $\mathbf{I}(t)$ behaves like a sort of running time average of **a**.

Similarly, in Fig. 3 quantities (2) and (9) are shown. The instantaneous reactive intensity $\mathbf{J}(t)$ appears to be the envelope signal of **r**, whose mean value is clearly zero as it oscillates around the zero line.

The two comparisons of the coupled alternative definitions (1) with (4) and (2) with (5) are shown respectively in Figs. 4 and 5. For both radiating and oscillating instanta-

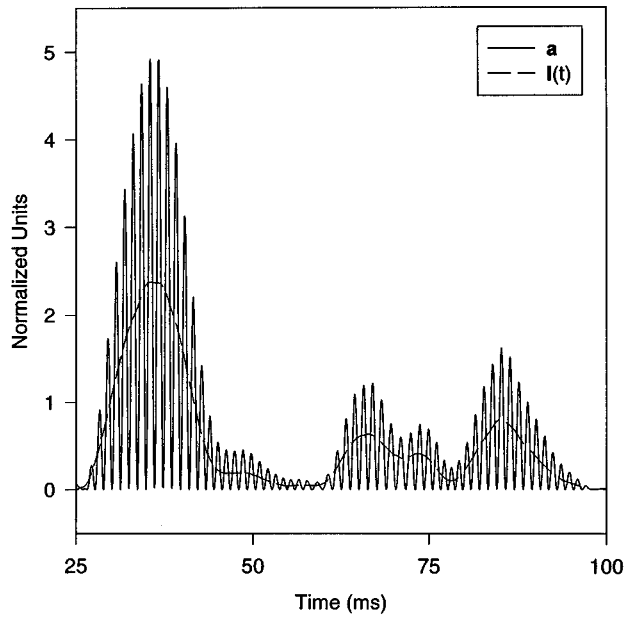

FIG. 2. Comparison of radiating and active instantaneous intensities in the same field conditions as Fig. 1.

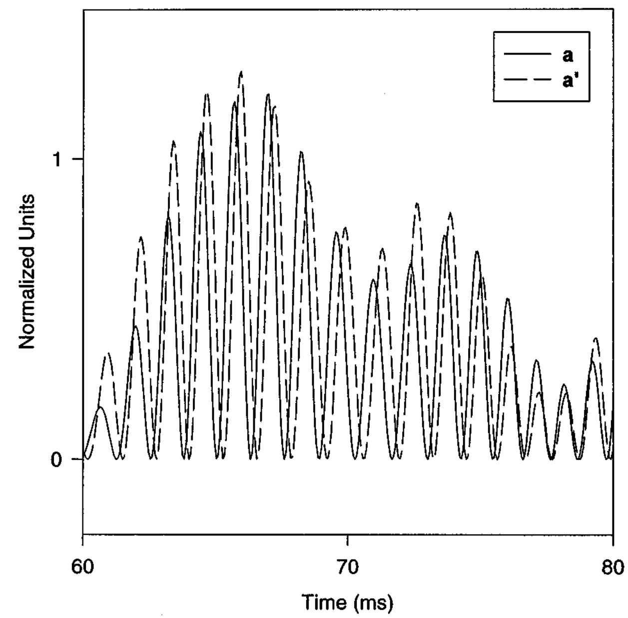

FIG. 4. Comparison of radiating intensity with its alternative definition in the same field conditions as Fig. 1.

neous fluxes, the two formulations give in this case a substantially equivalent time-dependent picture.

Next, measures have been taken in a reverberation room ($V=245\ \mathrm{m}^3$, $RT\simeq 4$ s) and are here presented in Figs. 6–11. Figure 6 refers to a pure tone at 800 Hz as stimulus, with a distance between the probe and loudspeaker of 1 m. In this case the relationships detected above, that is $\mathbf{I}(t)$ being a running time average of $\mathbf{a}$ and $\mathbf{J}(t)$ acting as the envelope signal of $\mathbf{r}$, are confirmed. It is also worth noting the sign of $\mathbf{J}(t)$, which is negative even if the probe is facing the speaker.

In the same room conditions, the stimulus was changed to a random noise in the third octave band centered at 630 Hz and the related measures are shown in Figs. 7–11. Figure 7 provides an interesting example of ‘‘cooperation’’ between pressure and normalized velocity (namely $\rho c\mathbf{v}$, where ρ is the equilibrium air density and c is the speed of sound): The zones where the two variables have the same time dependence supply exclusively radiating contribution to instantaneous intensity, whereas the rest gives an oscillating contribution. A link of this kind can be checked by considering Figs. 8 and 9 where the plots of $\mathbf{a}$ and $\mathbf{I}(t)$ and of $\mathbf{r}$ and $\mathbf{J}(t)$ are shown, respectively. In the present field conditions neither of the two relations of running the time average and the

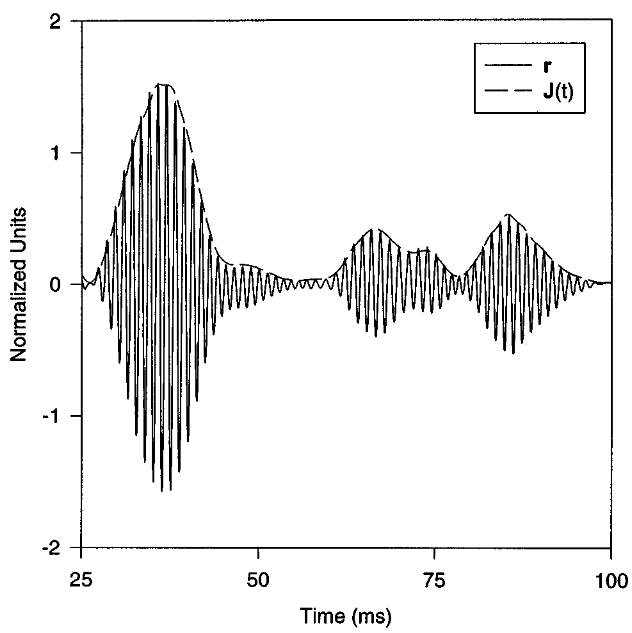

FIG. 3. Comparison of oscillating and reactive instantaneous intensities in the same field conditions as Fig. 1.

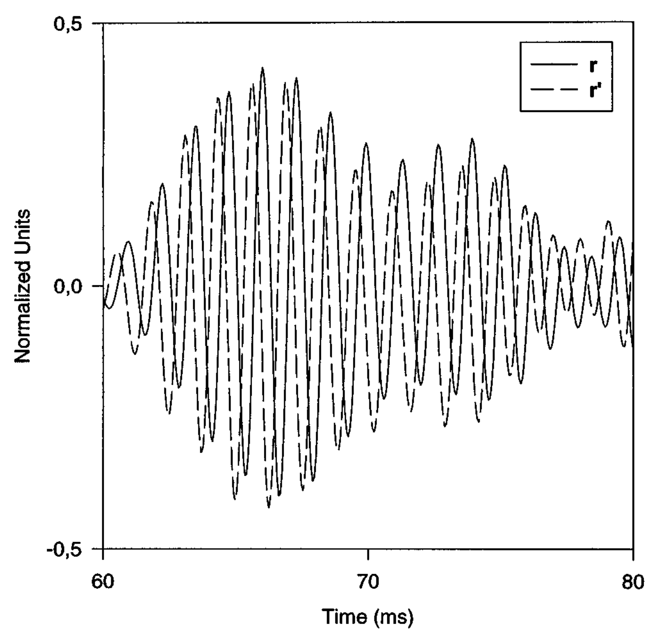

FIG. 5. Comparison of oscillating intensity with its alternative definition in the same field conditions as Fig. 1.

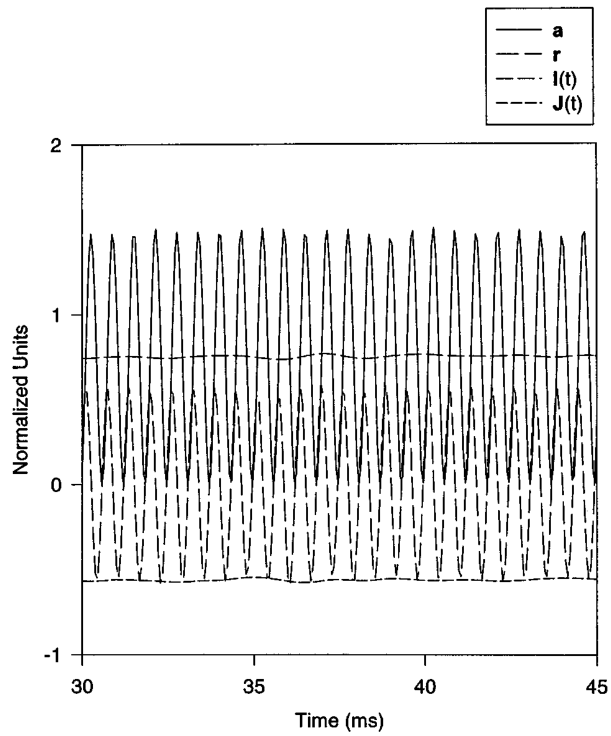

FIG. 6. Intensimetric quantities measured inside a reverberation room. Stimulus is a pure tone of 800 Hz and the distance between the loudspeaker and the facing probe is 1 m.

envelope signal between the newly defined quantities and the active and reactive instantaneous intensities encountered above is valid. Hence it is not possible to establish a straightforward connection linking the new intensimetric quantities

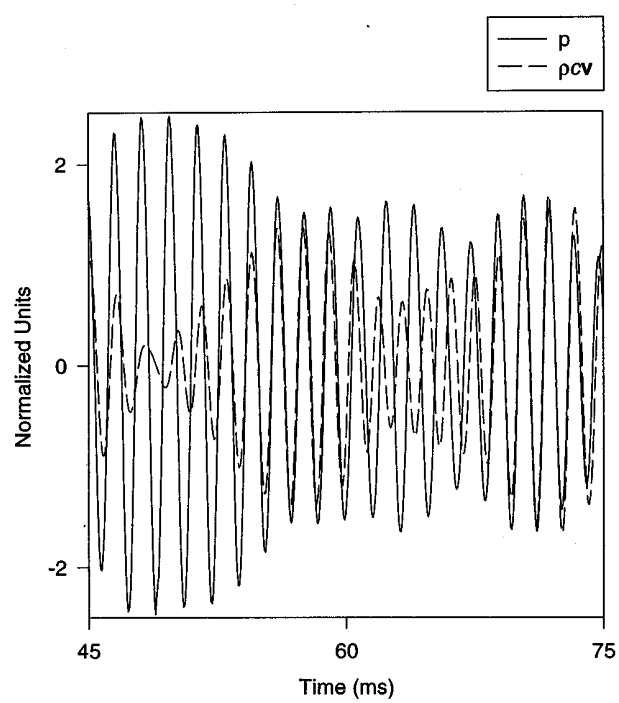

FIG. 7. Time histories comparison of pressure and normalized velocity inside a reverberation room. Stimulus is $\frac{1}{3}$ octave band filter noise with center frequency of 630 Hz and the distance between the loudspeaker and the facing probe is 1 m.

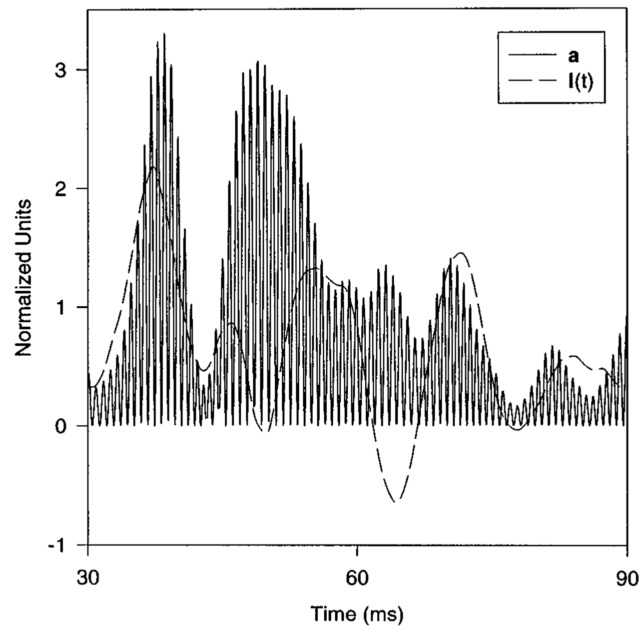

FIG. 8. Comparison of radiating and active instantaneous intensities in the same field conditions as Fig. 7.

to the real and imaginary parts of the complex intensity.

In Figs. 10 and 11 the alternative definitions $\mathbf{a}'$ and $\mathbf{r}'$ are characterized by a time-dependent picture which is completely different from that of **a** and **r**.

In addition to the time dependence of the above mentioned quantities, their spatial dependence was investigated through measurements of their time average values in different points inside test fields.

The measures were taken along the loudspeaker axis at various distances inside the anechoic room and each time average measurement at a fixed point was repeated 20 times and statistically analyzed. The loudspeaker was fed with a

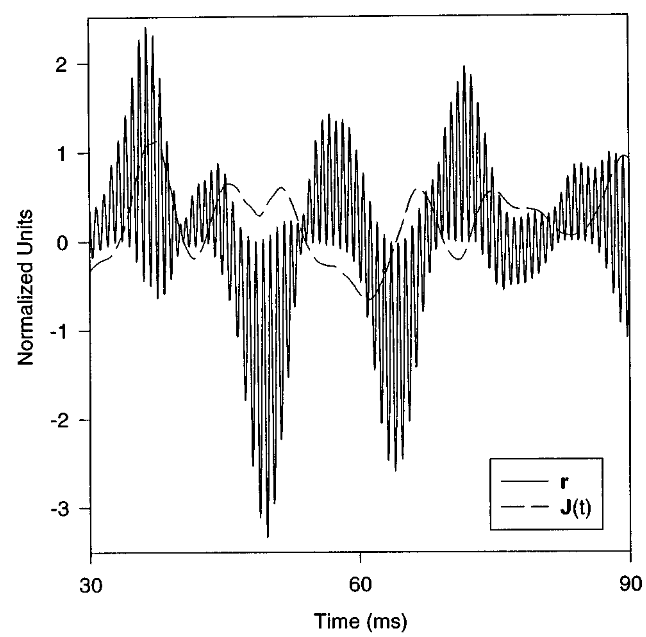

FIG. 9. Comparison of oscillating and reactive instantaneous intensities in the same field conditions as Fig. 7.

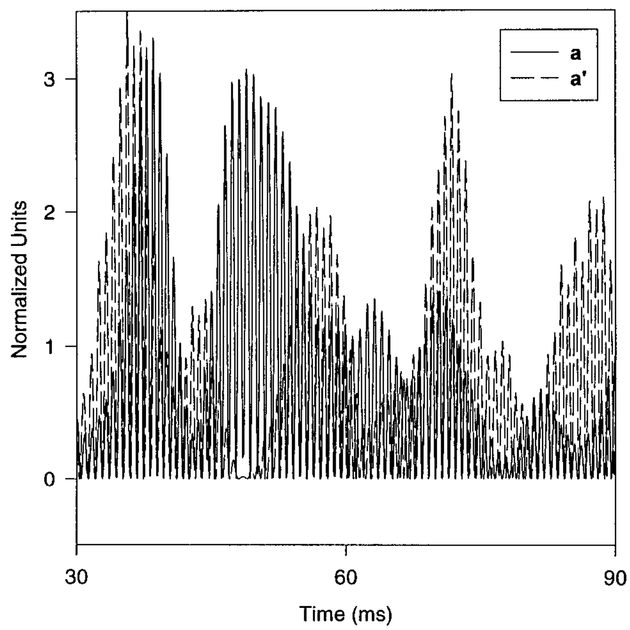

FIG. 10. Comparison of radiating intensity with its alternative definition in the same field conditions as Fig. 7.

third octave band filtered noise centered at 400 Hz. Figure 12 reports the comparison of the averages of the different definitions of radiating and oscillating intensities. Their respective values coincide within errors, thus validating Eqs. (6) and (7). Incidentally one can interpret the intersection point between radiating and oscillating intensity, occurring at about 25 cm from the loudspeaker, as the physical separation between the near field and the far field. In the near field there is a predominant effect of energy locally confined, whereas the far field is characterized by the stronger radiation of energy.

Then a second loudspeaker was added facing the previous one at a distance of 1.1 m and measures were taken along the line connecting the two. The loudspeaker on the right was fed with a pure tone of 385 Hz while the left one with an equal amplitude pure tone of 415 Hz. The choice of rather close frequency signals (both within the third octave band centered in 400 Hz) was intended to study the field in a quasi-interference condition. Figure 13 shows the time independent R and the modulus of reactive intensity $|\mathbf{J}|$ for this bichromatic superposition field. The two quantities behave in a completely different manner in the region of interest: R tells us that at the points close to the center a great amount of

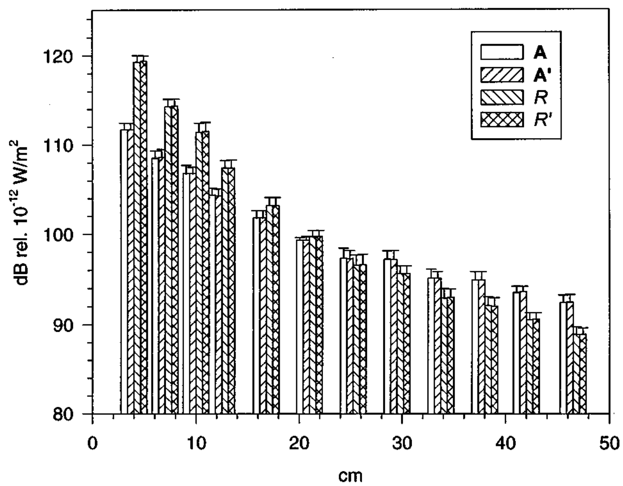

FIG. 12. Averaged intensimetric quantities measured in the region in front of a loudspeaker inside an anechoic room. Stimulus is $\frac{1}{3}$ octave band filter noise with center frequency of 400 Hz.

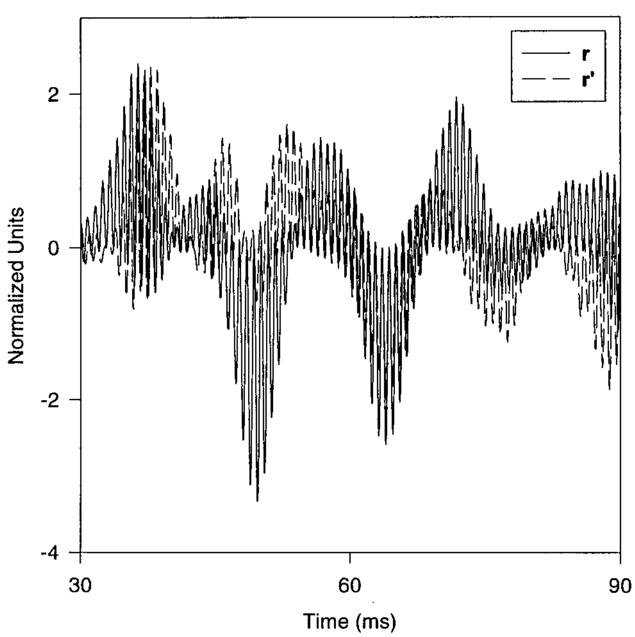

FIG. 11. Comparison of oscillating intensity with its alternative definition in the same field conditions as Fig. 7.

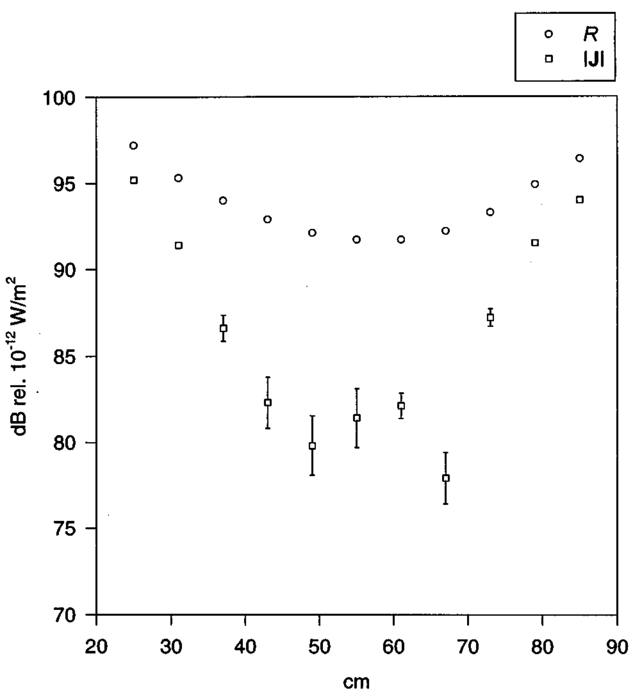

FIG. 13. Comparison of averaged oscillating intensity with the modulus of averaged reactive intensity in the region between two loudspeakers which are 1.1 m apart inside an anechoic room. The loudspeaker on the right is fed with a pure tone of 385 Hz and that on the left with a pure tone of 415 Hz with equal amplitude.

energy is indeed locally confined, just as it happens in the near field of a single loudspeaker. This result is consistent with the interpretation of progressive and standing wave fields within the proposed picture of the sound energy transfer. In detail, the more the condition of equal frequency is approached, the more the field in the region of interest resembles a standing wave pattern, which is agreed to be an acoustic field with all the energy trapped locally. The quantity $|\mathbf{J}|$ shows instead much lower values at the same points because the contributions coming from the two sources add vectorially with opposite signs.

IV. CONCLUSIONS

New definitions of instantaneous intensimetric quantities in general stationary fields called radiating and oscillating intensities have been presented and measured for some sound field conditions.

It has been experimentally shown that, although two alternative different time-dependent definitions are possible, they both have the same properties to the second statistical order.

The introduction of radiating and oscillating intensities allows the time average energy transfer process within general stationary fields to be described just in terms of the active intensity maps plus a scalar field accounting for energy which does not contribute to the net flow.

The comparison between instantaneous active and reactive intensity and the newly introduced quantities has shown that there is no straightforward general relationship between them, even if a kind of functional relation appears in some of the tested fields. At last, although the time average of the radiating intensity coincides with that of the instantaneous active intensity, the average properties of oscillating and reactive intensity are substantially different in a general (nonmonochromatic) field.

ACKNOWLEDGMENTS

The measurements presented in the paper have been taken at the Acoustics Laboratory of the Technical University of Denmark under the supervision of Professor Finn Jacobsen. The authors are grateful to Professor Jacobsen both for his support and for the fruitful discussions on intensimetric quantities. The authors wish to thank Professor G. Schiffrer of the Physics Department of University of Ferrara for the critical reading of the manuscript. This work has been supported by a grant from the ''Ferrara Ricerche'' research pool and by a grant from the National Research Council of Italy within the program ''Short term fellowship.''

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